Worksheet: What's Hard About Fraction Number Sense?

Type of Understanding or Knowledge	Example of Student Difficulty or Understanding	Tasks and Experiences That Build This Understanding (For your notes)
A Fraction is a Number • A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).	• When asked to put the fraction $\frac{2}{3}$ on a number line, a student said "you can't put it on a number line, because it's two pieces out of three pieces, it's not a number." (Or "$\frac{2}{3}$ is not a number, it's two numbers.")[i]	
Partitioning Fractions • A whole can be divided into smaller and smaller equal parts. • The same quantity can be represented by different fractions.	• Does not divide a whole into *equal* parts. • Difficulty seeing that $\frac{1}{2}$ is equal to $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ …and so on.	
The Meaning of the Denominator • Different units (such as $\frac{1}{3}$ and $\frac{1}{5}$) are different sizes. • The more units a whole is partitioned into, the smaller each one is. • $\frac{1}{n}$ fits exactly n times into the whole.	• Students add $\frac{1}{3} + \frac{1}{5}$ and get $\frac{2}{8}$, without realizing they are adding two different things (thirds and fifths), sort of like adding apples and hammers. • Students may think "$\frac{1}{5}$ is bigger than $\frac{1}{4}$ because 5 is bigger than 4." • Difficulty seeing that $\frac{1}{3}$ fits in the whole 3 times, $\frac{1}{4}$ fits in the whole 4 times. Difficulty seeing that $\frac{3}{3}, \frac{4}{4}$ etc. equal 1.	
Knowing What is the Whole Constructing the whole when given a fractional part. • Keeping track of the whole.	• Difficulty making the whole when you give them a fractional part, e.g.: "This paper is $\frac{2}{3}$; show me the whole." • Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g., half of a small cookie is not the same as half of a large cookie).	

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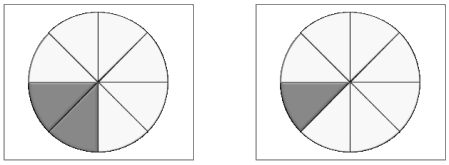
	• Confusion about whether the two drawings below together represent $\frac{3}{8}$ of a pie or $\frac{3}{16}$ of a pie. 	
Fraction Size • Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator - not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator. • Sees non-unit fraction as an accumulation of unit fractions. (A unit fraction has a numerator of 1; a non-unit fraction has a numerator other than 1.)	• May think $\frac{4}{9}$ is bigger than $\frac{3}{4}$ because 4 is bigger than 3 (comparing numerators), or $\frac{4}{9}$ is bigger than $\frac{3}{4}$ because 9 is bigger than 4 (comparing denominators), or $\frac{3}{5}$ is the same size as $\frac{5}{7}$ because the difference between the top and the bottom in both fractions is 2. • Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In $\frac{2}{4}$, $\frac{4}{8}$, $\frac{3}{6}$, etc. denominator is two times numerator. • Sees $\frac{5}{8}$ is made up of 5 eighths or 5 times $\frac{1}{8}$, that $\frac{9}{8}$ is made up of 9 eighths or 9 times $\frac{1}{8}$, etc.	
Fractions Can Represent Quantities Greater than One • May be difficult for students who have a strong image of a fraction as a *piece* of something.	• "You can't have $\frac{6}{5}$, because there's only $\frac{5}{5}$ in a whole."	

[i] Kerslake, D. (1986). *Fractions: Children's strategies and errors. A report of the strategies and errors in Concepts in Secondary Mathematics and Science Project*. Windsor, England: NFER-Nelson. Behr, M.J. & Post, T.R. (1992). Teaching rational numbers and decimal concepts. In T.R. Post (Ed.), *Teaching mathematics in grades K-8, research-based methods* (pp. 201-248). Boston: Allyn and Bacon.