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Focus on Learning Problems in Mathematics
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Initial Treatment of Fractions in Japanese Textbooks

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Abstract

Despite a long history of research and curriculum development efforts, fraction teaching and learning remains a major challenge for U.S. teachers and students. In contrast, according to the TIMSS, Japanese students appear to be very successful on problems involving fractions. Because textbooks play an important role in mathematics teaching and learning, 6 elementary school mathematics textbook series were analyzed for its treatment of fractions. The study investigated the following questions: (1) what are the specific fraction understandings the Japanese curriculum and textbooks attempt to develop and at what grade levels? (2) how do the Japanese textbooks introduce various fraction related ideas during Grade 4? and (3) what representations do the Japanese textbooks use as they introduce and develop fraction-related ideas during Grade 4?

The findings of the study raise some important questions for mathematics educators and curriculum developers in the United States: (1) Why do we introduce fractions so early in our curricula? (2) How can we intentionally support children's learning of fractions through careful selection of problems and representations? and (3) How can we help students go beyond the part-whole meaning of fractions? Is the notion of *measurement-fractions* potentially useful with U.S. students?

Research that looks across countries can provide a sharper picture of what matters in instruction aimed at developing proficiency.
(National Research Council, 2001, p. 358)

Despite a long history of research and curriculum development efforts, fraction teaching and learning remains a major challenge for U.S. teachers and students. Figure 1 presents some of the related items involving fractions from the Third International Mathematics and Science Study (TIMSS). As

Figure 1 indicates, U.S. students performed at or near the international average on these items, but their performance is far less than what we would like it to be. In contrast, more than 80% of Japanese students responded correctly to the same items. In fact, Japanese students outperformed their U.S. counterparts on all released items involving fractions. The Japanese curriculum introduces fractions later than typical U.S. curricula do (Watanabe, 2001a), thus, the Japanese students performed better than U.S. students in spite of an earlier and more frequent discussion of fractions in U.S. schools. This observation naturally raises the question, "How does the Japanese curriculum treat fractions?" In this paper, I will present the findings from a study that investigated the initial treatment of fractions in the Japanese na-

Item I-6

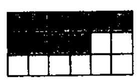
Write a fraction that is larger than $\frac{2}{7}$.

Answer: ____________________

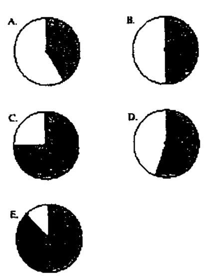
Item I-6

Int'l Avg.	75%
U. S.	81%
Japan	87%

Item K-1



Which circle has approximately the same fraction shaded as that of the rectangle above?



Item K-1

Int'l Avg.	70%
U. S.	62%
Japan	81%

Item L-17

What is the value of $\frac{2}{3} - \frac{1}{4} - \frac{1}{12}$?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{3}{8}$

D. $\frac{5}{12}$

E. $\frac{1}{2}$

Item L-17

Int'l Avg.	50%
U. S.	51%
Japan	80%

Item N-14

In which list of fractions are all of the fractions equivalent?

A. $\frac{3}{4}, \frac{6}{8}, \frac{12}{14}$

B. $\frac{3}{5}, \frac{5}{7}, \frac{9}{15}$

C. $\frac{3}{8}, \frac{6}{16}, \frac{12}{32}$

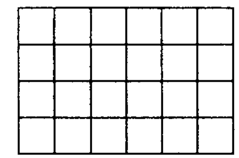
D. $\frac{5}{10}, \frac{10}{15}, \frac{1}{2}$

Item N-14

Int'l Avg.	67%
U. S.	67%
Japan	82%

Item N-19

Shade in $\frac{5}{8}$ of the unit squares in the grid.



Item N-19

Int'l Avg.	52%
U. S.	43%
Japan	80%

Figure 1 Some of the TIMSS Grade 8 released items and U.S. and Japanese students' performance.

Figure 1. *Some of the TIMSS Grade 8 released items and U.S. and Japanese students performance.*

tional curriculum and their elementary school mathematics textbooks. The purpose of the study was to provide a detailed description of the way fractions are treated in the Japanese textbook series. It is hoped that such a description may facilitate a critical reflection on the way fractions are treated in U.S. curricular materials

Why study a curriculum and/or textbooks?

Clearly, many factors influence how teachers teach mathematics in their classrooms. As Stigler & Hiebert (1999) noted, teaching is a cultural activity in which teachers follow their cultural scripts. As a cultural activity, no single factor will explain why teachers in a particular way. Nevertheless, understanding how various factors influence classroom teaching, either singularly or in combination, should provide some valuable insights for the mathematics education community in the United States.

One critical factor that influences teaching and learning of mathematics is the curriculum (Schmidt, McKnight, & Raizen, 1996). The curriculum analysis conducted within the TIMSS framework suggested that a typical U.S. curriculum is unfocused, undemanding, and incoherent (Schmidt, Houang, & Cogan, 2002). The analysis by Schmidt et al. (2002) shows that the high-performing countries' curricula tend to be more focused (fewer topics in each grade level), cohesive (logical sequencing of mathematical topics) and with higher mastery expectations (much less repetition of the topics across grades).

Of course, investigating the nature and quality of a curriculum in any country is a complicated matter. In the Second International Mathematics Study, the International Association for the Evaluation of Educational Achievement (IEA) considered three "faces" of curriculum – the intended, implemented, and attained. The intended curriculum is the curriculum established at the system level. In Japan, the Ministry of Education, Culture, Sports, Science and Technology (the Ministry hereafter) publishes the Course of Study (COS), which specifies the content goals and time allocation for each subject matter. In addition, the Ministry publishes a series of commentary books for each subject matter at each level (elementary, lower secondary, and upper secondary) to articulate the points of consideration regarding the content, instructional approach, and assessment. Through these documents, the Ministry makes public the intended curriculum.

Clearly, the intended curriculum influences the actual classroom instruction, that is, the implemented curriculum, but its influences are not always direct. Textbooks and accompanying teachers' manuals play an important role. Schmidt et al. (1996) consider these materials as a "potentially implemented curriculum" (p. 30), and their role is to bridge between the intended and implemented curricula. Shimahara and Sakai (1995) report that significant numbers of both American and Japanese elementary school teachers rely heavily on teachers' manuals as they teach mathematics. According to one Japanese college-level mathematics educator, about 70% of elementary school teachers rely on teachers manuals when they teach mathematics lessons (Shigematsu, personal communication, April, 1997). Japanese mathematics educators sometimes lament mediocre teachers simply holding the teachers' manual and teaching directly from the book. If it is indeed the case that both American and Japanese teachers rely on teachers' manuals to conduct their mathematics lessons, at least a part of the reason for the different nature of mathematics lessons in the U.S. and in Japan might be attributable to the way teachers' manuals are organized. Watanabe's (2001b) analysis of the overall structure and contents of teachers' manuals in Japan and the United States does reveal significant differences, and further investigation along this line may provide new insights into the curricular and achievement differences, that is, the differences in intended, implemented and attained curricula, between the U.S. and Japanese students.

Students' understanding of fractions

Typically, simple fractions such as one half, one third, and one fourth are introduced as early as kindergarten in the U.S. More formal instruction

on fractions, including ideas such as comparing fractions and equivalent fractions, usually takes place during the early intermediate grades, around Grades 3 or 4. Students then move on to computation with fractions starting as early as Grade 4 or 5. Despite such an early introduction and repeated treatment of fractions, many upper elementary school students' understanding of fractions leaves much to be desired. Consider the following example from Simon (2002).

> In a fourth-grade class, I asked the students to use a blue rubber band on their geoboards to make a square of a designated size, and then to put a red rubber band around one half of the square. Most of the students divided the square into two congruent rectangles. However, Mary, cut the square on the diagonal, making two congruent right triangles. The students were unanimous in asserting that both fit with my request that they show halves of the square. Further, they were able to justify that assertion.
>
> I then asked the question, "Is Joe's half larger; is Mary's half larger, or are they the same size?" Approximately a third of the class chose each option. In the subsequent discussion, students defended their answers. However, few students changed their answers as a result of the arguments offered.
>
> (Simon, 2002, p. 992)

In an earlier study (Watanabe, 1995), similar questions were posed to 16 fifth graders in individual interviews. Congruent squares were cut into two equal parts in three different ways: by a vertical line, by a diagonal line, and by a slanted line that created two congruent trapezoids. (See Figure 2.) After the students verified that two copies of each shape were identical and they could be put together to form the same square, they were given one of each shape and asked, "If these were cookies and you were really hungry which one would you pick?" All but two students initially picked one of the three to be the largest. Even after they were reminded of the initial demonstration that two copies of each shape made up congruent squares, 8 of those students maintained that the piece they selected was the largest. Simon (2002) concluded that these students had the understanding of fractions as an arrangement rather than a quantity.

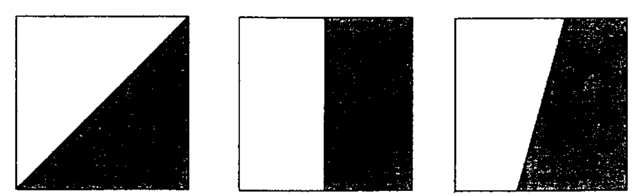


Figure 2. *Three congruent squares were partitioned into two equal parts in three different ways.*

Fraction teaching and learning have been a focus of research for a long time. Kieren (1980) identified 5 sub-constructs of fractions: part-whole, operator, quotient, measure, and ratio. A variety of research projects, both large and small scale, utilized these sub-constructs in their studies of teaching and learning of fractions. Probably, the most extensive study of fractions was carried out under the Rational Number Project (e.g., Hehr, Harel, Post & Lesh, 1992, 1993). Other researchers have also taken advantage of the notion of fraction sub-constructs in their studies. Many studies provided detailed descriptions of the challenges students faced as they attempted to solve problems involving fractions. One consensus that seems to emerge from these studies was that children's whole number understanding interfered with their effort to make sense of fractions: for example 1/3 is greater than 1/2 because 3 is greater than 2. Such difficulty creates a major challenge for teaching of fractions. Two other examples of challenges students face were cited by Larson's (1980) study revealing challenges in locating a fraction on a number line and by Greer's (1987) study reporting challenges in selecting an appropriate operation when problems involved rational numbers.

Mack (1990, 1995) investigated children's informal understanding of fractions and how it might be utilized in formal fraction instruction. In particular, she suggested that a sequence of instruction which begins with partitioning of a whole and then expanding to include other strands might be effective. Pothier and Sawada's (1983, 1989) work shows that there is a pattern in young children's development of partitioning strategies and justifications for equality of parts. Armstrong and Larson (1995) investigated how students in fourth, sixth and eighth grades compared areas of rectangles and triangles embedded in another geometric figure. They found that although most students used direct comparison methods, explanations based on part-whole, or partitioning, increased as students became more familiar with fractions. These studies suggest the importance of partitioning activities in the beginning of fraction instruction. Unfortunately, most textbook series provide children pre-partitioned figures. As a result, children themselves do not engage in the act of partitioning, and those activities become simply counting activities for children.

More recently, Steffe, Olive, Tzur and their colleagues have embarked upon an ambitious study to articulate children's construction of fraction understanding (e.g., Olive, 1999, Steffe, 200; Tzur, 1999, 2004). The *reorganization hypothesis* (Olive, 1999) offers an alternative perspective on teaching and learning of fractions. According to their findings from a teaching experiment, children's whole number concepts did not interfere with their efforts to make sense of fractions (Olive, 1999, Steffe, 2000; Tzur, 1999, 2004). In fact, the types of units and operations children constructed in their whole number sequence can facilitate their reorganization of fraction schemes. However, the nature of instruction and the types of problems

used in instruction, not limited to fraction instruction but also including instruction on multiplication, division, and so on, must be carefully aligned with such a potential development of fraction understanding. For example, multiplication is often considered as simply repeated addition. Although repeated addition is a tool to calculate the product, multiplication is much more than repeated addition. Rather, students should be encouraged to understand multiplication as a way to quantify something when it is composed of several copies of identical size, and this is exactly what is emphasized in the Japanese curriculum (Watanabe, 2003). Such an understanding can become the basis of understanding fraction $\frac{m}{n}$ as m times of $\frac{1}{n}$, instead of "m out of n," which does not necessarily signify a quantity. Thompson and Saldanha (2003) noted that "we rarely observe textbooks or teachers discussing the difference between thinking of 3 as 'three out of five' and thinking of it as '$\frac{3}{5}$ one fifth'" (p. 107).

What this brief review of research literature suggests is that the research findings have not significantly influenced the textbook treatment of fractions in the United States. In fact, in some cases, the textbook treatment of fractions go counter to the research findings. Perhaps an in-depth study of how fractions are treated differently in another country's textbook series may serve as a catalyst to re-elevate the way fractions are typically treated in the U.S. textbooks.

Research questions

The overall research goal was to gain a better understanding of how fractions are introduced and developed in the Japanese curriculum and textbooks. For the analysis of the textbook treatment, I focused my analysis on Grade 4, the year when fractions are first introduced and discussed. Specifically, the study tries to answer the following questions:

- What are the specific fraction understandings the Japanese curriculum and textbooks attempt to develop and at what grade levels?
- How do the Japanese textbooks introduce and develop various fraction related ideas during Grade 4?
- What representations do the Japanese textbooks use as they introduce and develop fraction-related ideas during Grade 4?

The first question was intended to help us understand if and how the Japanese elementary mathematics curriculum and textbooks incorporated the findings from the existing research. For example, does the Japanese curriculum and textbooks treat non-unit fractions as iteration of a unit fraction? The last two questions primarily focused on the way the curriculum and textbooks might support students' learning of fractions.

Methodology

The National Course of Study (Japan Society of Mathematical Education, 2000) and *Commentary on the National Course of Study: Elementary School Mathematics* (Ministry of Education, 1999) were included in the analysis of the Japanese national curriculum. Because the treatment of fractions in the Japanese curriculum is completed in Grade 6, the final year of their elementary schools, only the *Commentary* for elementary school mathematics was included in the analysis. These documents were the primary sources to answer the first research question although the textbooks and accompanying teachers' manuals were also included in the analysis. The documents were analyzed first to identify the timing and the specific focus of the curricular treatment of fractions in each grade level. In addition to noting the timing of fraction instruction, the analysis attempted to locate the fraction instruction in relationship to other relevant mathematical ideas. Those mathematical ideas are multiplication and division operations with whole numbers, decimal numbers, and measurement.

Since the two government documents only identify and explain the specific learning expectations but not how they should be accomplished, textbooks were analyzed to answer the last two research questions. There are six commercially published textbook series for elementary school mathematics that have been approved by the Ministry. For the textbook treatment of fractions, I focused my analysis on how fractions are initially introduced and developed. Since this takes place, according to the national curriculum documents, in grade 4, my analysis focused on Grade 4 textbooks. The Grade 4 pupils' books for all six series were included in the analysis. Furthermore, the teachers' manual accompanying the most widely used series was also included in the analysis.

Watanabe (2001a) reported that the Japanese textbooks are organized so that each lesson will focus on one (or a few) problem(s). Therefore, to analyze the textbooks, I have focused on the following two specific aspects: (a) the nature of the problems, that is, is the problem contextualized or presented purely symbolically, and if problems are contextualized, what is the context, and (b) the type of representation used, that is, does the textbook use any non-symbolic representation, and if so, what types.

Findings

Learning Goals

The *Commentary* specifies the learning goals with respect to fractions very explicitly. Table 1 summarizes the fraction related topics discussed in the Ministry of Education documents. As the table shows, fractions are not formally introduced in the Japanese curriculum until Grade 3. In many textbooks in the United States, simple fractions such as 1/2, 1/3 and 1/4 are included starting with Grade 1 (e.g., Clements, Jones, Moseley & Schulman, 1999). Therefore, the Japanese curricular treatment of fractions starts much

later than is the case in a typical U.S. curriculum. On the other hand, fractions are prominently discussed in middle school mathematics textbooks in the United States (e.g., Larson, Boswell, Kanold, & Stiff, 1999). Therefore, the Japanese curricular treatment of fractions is much more concentrated with a clear mastery expectation by the end of Grade 6.

Table 1. *Summary of fraction related topics discussed in the Ministry of Education documents.*

Grade 4	Introduction of fractions; improper fractions and mixed numbers; comparison of fractions (with like denominators only)
Grade 5	Comparison of fractions (unlike denominators); equivalent fractions; addition and subtraction of fractions with like denominators; fractions as quotient; relationships among fractions, decimals & whole numbers
Grade 6	Addition and subtraction of fractions with unlike denominators; creating equivalent fractions; multiplication and division of fractions

Prior to the study of fractions, students have completed the study of whole number multiplication (in Grade 3) and (in Grade 4) the study of whole number division, which included division by 2- or 3-digit numbers and the division algorithm relationship,

Dividend = Divisor x Quotient + Remainder.

Decimal numbers are introduced in Grade 4; however, the Ministry documents do not specify whether decimals or fractions should be discussed first. Of the 6 elementary school mathematics textbooks, only one series introduces fractions prior to discussing decimal numbers. The scope of the Grade 4 discussion of decimal numbers is limited to the first decimal place (or $\frac{1}{10}$'s place). Addition and subtraction of decimal numbers are also discussed in Grade 4. Multiplication and division of decimal numbers are discussed in Grade 5, when the Japanese COS completes the treatment of decimal numbers.

Table 2 summarizes the content of the measurement strand in the Japanese COS. As the table shows, before the introduction of fractions, the Japanese curriculum completes the study of measurements on the following attributes: length, capacity and weight. In Grade 4, the same year children begin their investigation of fractions, the area measurement is also introduced. Since the COS does not specify the order of topic within a given grade level, the order in which these topics are treated in a textbook varies. Of the six textbook series, three, including the two most widely used series, discuss the area measurement prior to the introduction of fractions, while the other three introduce fractions prior to their discussion of the area measurement. The fact that the area measurement is also a new concept in Grade 4 may have some impact on the types of models used in these textbook series, as it will become clearer later.

Table 2. *Summary of the measurement strand in the Japanese COS*

Grade	Content
1	Introduction of length – direct and indirect comparison, the use of informal units
2	Linear measurement with the units of m (meter), cm (centimeter) and mm (millimeter). Clock reading
3	Linear measurement with the unit of km. Introduction of capacity and weight, using the units of l (liter) and g (gram), respectively. Other units of capacity (milliliter and deciliter) and weight (kilogram) are also touched upon.
4	Introduction of area measurement using the units of cm2 (square centimeter). Calculating the area of squares and rectangles. Introduction of angle measurement using the unit of degree.
5	Area of plane figures, including triangles, parallelograms, and circles.
6	Introduction of volume, using the unit of cm3 (cubic centimeter), and calculating the volume of rectangular prisms (cubes and cuboids).

Meanings of fractions

According to the *Commentary,* there are five different meanings of fractions discussed in the elementary school mathematics curriculum. Those meanings are, using the fraction $\frac{2}{3}$ as an example,

1. two parts of a whole that is partitioned into three equal parts
2. representation of measured quantities such as $\frac{2}{3}\,l$ or $\frac{2}{3}\,m$
3. two times of the unit obtained by partitioning 1 into 3 equal parts
4. quotient fraction $(2 \div 3)$
5. A is $\frac{2}{3}$ of B – if we consider B as 1 (a unit), then the relative size of A is $\frac{2}{3}$.

According to the *Commentary,* Grade 4, when fractions are first introduced, the focus is on the first three meanings of fractions, while the quotient fraction becomes a focus in Grade 5. Fractions as ratio, the fifth meaning, are investigated in Grade 6 as students study proportions.

In the teachers' manuals, these five meanings are also discussed and elaborated. However, in the textbooks, the first two meanings are often combined together. In other words, many problems found in the Japanese textbooks are put in the context of measurement, where the whole is one measurement unit. Thus, the length equivalent to two of the three equally partitioned parts of 1 *meter* is described as "$\frac{2}{3}$ of 1 *meter,*" and the length is denoted as $\frac{2}{3}$ *m*. However, the primary role of the part-whole meaning of fraction seems to be the establishment of unit fractions, such as $\frac{1}{3}$ (or $\frac{1}{3}$ *m*). As the unit progresses, the textbooks place much more emphasis on treating a non-unit fraction as a collection of unit fractions, the third meaning of fraction in the *Commentary.* Thus, they will pose questions such as, "What are the lengths equivalent to two, three, or four $\frac{1}{3}$ *m*?" This meaning of fractions is then used to expand the range of fractions beyond proper fractions. Diagrams similar to Figure 3 are often included in the textbooks.

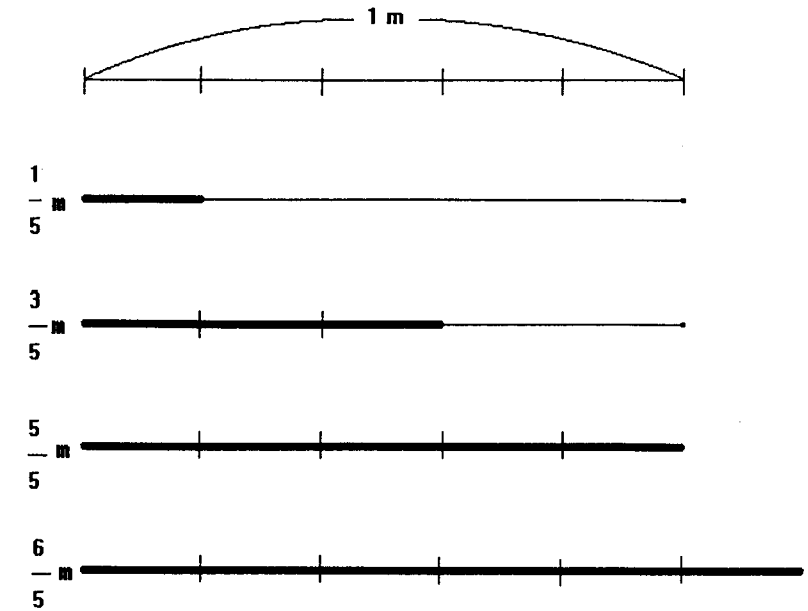


Figure 3. *Textbooks include a diagram like this to show how non-unit fractions are composed of unit fractions.*

The teachers' manual accompanying the most widely used elementary mathematics textbook suggests that the two main ideas about fraction concepts are (1) fractions are useful to denote the quantity less than 1 unit, and (2) fractions are numbers just like whole numbers and decimal numbers. The manual also states that the advantage of fractions is that we can flexibly establish new fractional units, but this flexibility poses a challenge of representing fractions on a number line.

Problems used in introducing and developing fractions concepts

What kinds of problems do the Japanese textbooks use to introduce and develop fraction concepts? Sugiyama, Iitaka and Itoh (2002) introduce fractions through a problem set using the context of a child measuring the circumference of a tree by wrapping a strip of paper around it. The picture of the paper strip shows that the circumference is slightly longer than 1 meter, and the question posed to students is how to express the length beyond 1 meter. Three other series use similar problems that are set in the context of linear measurement. One series (Nakahara, 2002) uses a liquid measure context instead, and one series (Hiraoka & Hashimoto, 2002) introduces fractions by asking the size of a piece of cake obtained by cutting the cake into two equal parts. Table 3 summarizes the problem contexts in the 6 textbook series.

Table 3. *Summary of problem contexts*

	A	B*	C	D*	E*	F
% of measurement problems in the fraction unit or units	98 %	60 %	40 %	38 %	45 %	51 %
% of problems shown with number line	0 %	13 %	21 %	38 %	27 %	21 %
% of problems presented only with symbols	2 %	26 %	39 %	23 %	21 %	28 %

A: Ichimatsu, Okada & Machida (2002)
B: Sugiyama, Iitaka & Itoh (2002) * Does not add up to 100 % due to rounding errors.
C: Hosokawa, Nohda, Shimizu & Funakoshi (2002)
D: Nakahara (2002) * Does not add up to 100 % due to rounding errors.
E: Hiraoka and Hashimoto (2002) * 7% of problems involved area measurement.
F: Sawada (2002)

There are two notable features of the way the Japanese textbooks introduce and develop fractions. First, of the six textbook series, five of them use opening problems that are set in a "mixed number" situation, that is, the fractional quantity investigated is a part of a quantity greater than one unit. This is true even of the one series that splits its treatment of fractions into two sections: fractions less than one and fractions greater than one. The only exception to this approach is Hiraoka and Hashimoto (2002) where the opening problem asks students how they might describe the size of a piece of cake obtained by cutting the original into two equal pieces. Problems of this nature seem to be much more common in U.S. textbooks. The use of mixed number contexts in the opening problems is consistent with the emphasis in the *Commentary* that fractions are useful to express those quantities that are less than one unit. Moreover, by using fractional amounts that cannot be expressed by a decimal number with one decimal place (e.g., $\frac{1}{3}$ and $\frac{1}{4}$), the textbooks demonstrate the flexibility of fractional units, another point emphasized by the *Commentary*.

Another feature of the problem used in the Japanese textbooks is that the measurement contexts used in the problems are either linear or liquid measurement. In fact, the only problems involving measurement other than length or capacity are the two opening problems involving measurement other than length or capacity are the two opening problems from Hiraoka and Hashimoto (2002) that involved partitioning of a cake. Even in this particular textbook, of the 33 problems in the unit, 11 involved linear measurement contexts while 4 additional problems involved liquid measurement. Table 4 summarizes the frequency of various measurement problems appearing in the six textbook series analyzed.

Representation

There are several different graphical representations that can be used to model fractions. The three most common models are area models, linear models, and discrete models (see Figure 4).

Table 4. *Summary of measurement problems in the textbook series.*

	A	B	C	D	E	F
% of measurement problems in the fraction unit or units	98 %	58 %	40 %	38 %	45 %	51 %
% of linear measurement problems among all measurement problems	72 %	59 %	69%	30 %	64 %	63 %
% of liquid capacity problems among all measurement problems	28 %	41 %	31 %	70 %	25 %	37 %

A: Ichimatsu, Okada & Machida (2002)
B: Sugiyama, Iitaka & Itoh (2002)
C: Hosokawa, Nohda, Shimizu & Funakoshi (2002)
D: Nakahara (2002)
E: Hiraoka and Hashimoto (2002)
F: Sawada (2002)

Unlike most U.S. textbooks, in which area models are the most dominant graphical representation for fractions, linear models are the primary graphical representations of fractions in the Japanese Grade 4 textbooks. Although the diagrams accompanying a liquid measure problem (see Figure 5) are similar to area models, they are different in the sense that they are much more context-bound. Therefore, it is not appropriate to share in the top 3 segments in Figure 4 because liquid cannot be floating inside a measuring cup.

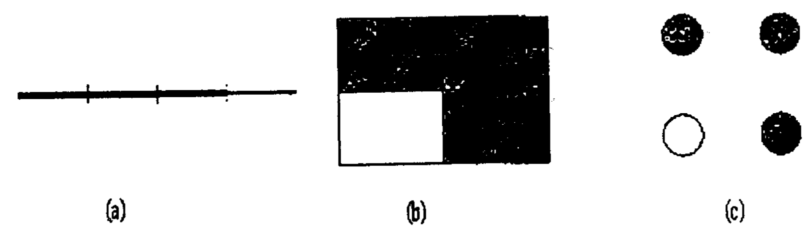


Figure 4. *Fraction ¾ represented using (2) linear model, (b) area model, and (c) set model.*

One of the reasons for not using area models to represent fractions appears to be the fact that the area measurement is introduced after the initial discussion of fractions. Although this was the case in only 3 of the six textbook series, there is also an historical factor. Unlike the most recent revision of the National Course of Study, which went into effect in the 2003-2004 school year, fractions were introduced in Grade 3 while area measurement was introduced in Grade 4. Therefore, under the previous COS, fractions were introduced before area measurement in all textbook series. Therefore, it is not surprising that textbook series, even if they now introduce area measurement prior to the introduction of fraction concepts, choose not to utilize unfamiliar representations in this particular context.

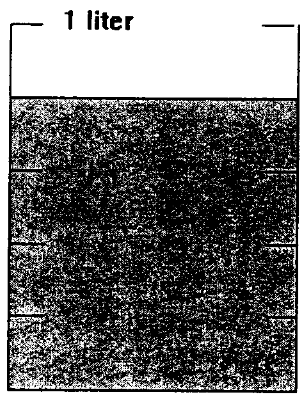


Figure 5. *A graphical representation like this often accompanied liquid measurement problems.*

Perhaps a much more significant reason for focusing on linear models is the Japanese curriculum's effort to establish fractions as numbers through the use of number line. Students are familiar with number lines as a representation of whole numbers and decimal numbers (except for those students who use Sawada & Okamoto (2002), which introduces fractions before decimal numbers). By representing fractions on a number line, the Japanese curriculum tries to help students view fractions as numbers. Toward this end, textbooks often include graphical representations that are very similar to number line like the one shown in Figure 6.

Furthermore, some textbooks will include graphical representations similar to the one shown in Figure 7 to intentionally connect the number line model with familiar representations of fractions.

Figure 6. *A diagram like this is used as a precursor to the number line representation of fractions.*

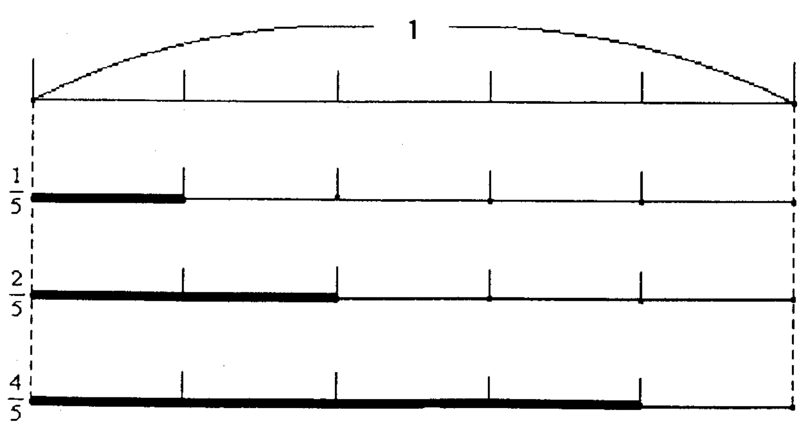
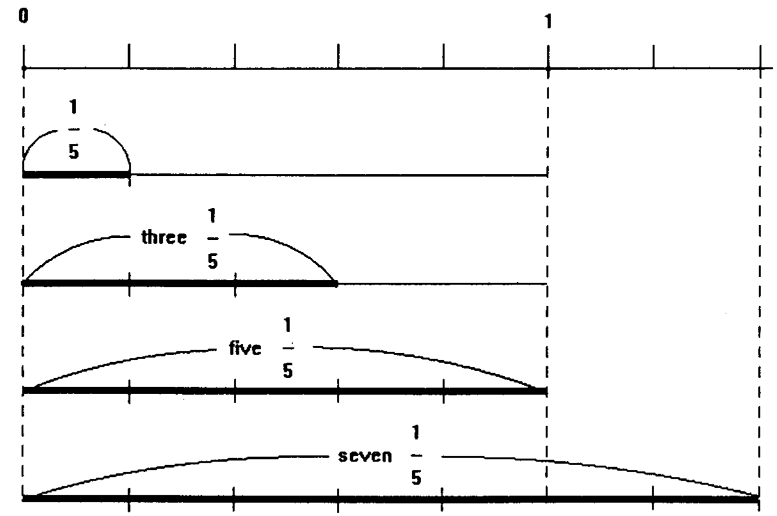


Figure 7. *Connecting fraction models (linear) to the number line. Note that there is no measurement unit in this representation unlike the one in Figure 4.*



These graphical representations are similar to the ones that represented the linear measurement problem contexts, thus they are familiar to children; however, they do not include a measurement unit, emphasizing that this is a representation for numbers.

Discussion

So, what do these findings tell us about the way the Japanese elementary mathematics curriculum introduces and develops fraction concepts? In terms of the timing of fraction introduction, the Japanese curriculum definitely introduces fractions later than typical U.S. textbook series do. However, the difference in the curricular treatments of fractions is not limited to the timing of its introduction. Perhaps more significantly, the Japanese elementary mathematics curriculum seems to progress through various fraction-related ideas with more focus and mastery expectations, as suggested by Schmidt et al's (2002) analysis of the overall mathematics curriculum. Thus, after 3 years, and about 47 lessons according to the suggested pacing of one series, the Japanese curriculum claims to have completed the study of fractions. This seems to be in stark contrast with the way fraction concepts are often developed (or not) in the U.S. textbooks. Typically, children in the U.S. are introduced to simple unit fractions with the denominators of 2, 3 and 4 in their first exposure with fractions. Then, the textbooks expand the scope of their treatment to include non-unit fractions and fractions with larger denominators. However, throughout this development, which may take place over a few grade levels, the meaning of fractions seems to stay constant – part of a whole. As Thompson and Saldanha (2003) note, rarely do we see in the U.S. textbooks a treatment of non-unit fractions as collections of unit fractions – a meaning emphasized in the introductory unit in the Japanese curriculum.

Another way Japanese textbook series are more intentional and purposeful is in their choice of representations. As discussed above, the Japanese textbooks appear to make an intentional effort to help students connect the linear representation of fractions with the number line. Furthermore, this emphasis on number lines and linear models may be one of the reasons for focusing on linear measurement as the problem context used when students were introduced to fractions. By pictorially representing the problem situations, the textbooks can naturally introduce the linear model of fractions. These linear models, then, are intentionally connected to the number line model. Moreover, representing quantities using a "tape diagram" is something students are familiar with from their earlier studies. Thus, students are introduced to a new concept within a familiar representation context.

These findings seem to raise several questions about the way fractions are treated in many U.S. textbooks. I will conclude this paper by discussing some of those questions. It is my hope that this article will begin a serious discussion on these issues.

Why do we introduce fractions so early in our curricula? It is clear that although the Japanese students are introduced to fractions later than the U.S. students are, their achievement level is higher at Grade 8. What do we gain by introducing fractions so early? Would U.S. students do even worse if they were introduced to fractions later? Is it possible that focusing primary grades mathematics instruction on fewer mathematical ideas would help them develop a deeper understanding of those ideas? Could that eventually improve their learning of fractions when they do encounter fractions.

Why do we place so much emphasis on area models? Is the 'pizza model' really helpful for children to understand fractions as numbers? Clearly, many children (and adults) can relate very easily to the 'pizza' or 'pie' model of fractions. However, does it make sense to focus so much of our attention on this model? How exactly is the 'pizza model' helpful for students' learning of various fraction-related ideas? Is it possible that the benefit of familiarity is outweighed by the challenges this circular area model poses? Furthermore, one of the reasons why we introduce fractions so early is that fractions are needed in the customary measurement system, in particular in linear and liquid measure contexts. However, if that is indeed the case, an intentional connection to linear and liquid measurement contexts seem to be much more needed in the U.S. classrooms than it is in Japan. Familiarity is an important consideration, but so is the connection within mathematics.

What are the strengths and weaknesses of various fraction models? Is it always better to use multiple models, or is it more helpful if instruction focuses on one particular model? Related to the previous question, we should investigate how other models might be helpful for children learning various fraction ideas. We need to understand not only how each model might be helpful but also what students need to understanding prior to using that model. How do young children who have yet to explore the concept of area make sense of this model? Whether one uses a circular region or not, when the area model is used, students will have to partition a geometric figure. What kinds of experiences with geometric shapes should children have to support their fraction learning using such models?

How can we intentionally support children's learning of fractions through careful selection of problems and representations? Number lines are something U.S. textbooks often use, but what challenges do students face when they use number lines to represent fractions? How can fraction instruction be designed so that we can help students deal with those challenges head on? Alternatively, how should we teach number lines with whole numbers so that children can use number lines as a tool to think about fractions as numbers?

How can we help students go beyond the part-whole meaning of fractions? Is the notion of 'measurement fractions' potentially useful with U.S. students? When number lines are used to represent fractions, there is an underlying assumption that fractions are numbers. However, when students' understanding of fractions is limited to the part-whole meaning, it is doubtful that they understand fractions as numbers. As the students in the quotation from Simon (2002) show, it is not uncommon for students to have a more qualitative understanding of fractions than a quantitative understanding. How can we organize our instruction so that we can facilitate children's development of an understanding of fractions as numbers? Could the notion of a 'measurement fraction' used in the Japanese curriculum be potentially useful? Could such an approach be helpful to support children's development of iterative understanding of non-unit fractions, that is, $\frac{a}{b}$ means a copies of $\frac{1}{b}$, which some studies seem to suggest beneficial (Olive, 1999, Stefee, 2002; Tzur, 1999, 2004)?

Conclusion

This study was conducted to provide an in-depth description of how fractions are treated in the Japanese elementary school mathematics curriculum. Although the article started with the data from the TIMSS showing a superior performance by the Japanese 8th graders compared to their U.S. counterparts, this study was not conducted to be an evaluative study. Rather, I hope that by understanding deeply how fractions are introduced and developed in another country, I can raise some questions about our current practice. It is my hope that a critical reflection on our current practices will help us improve both the quality of curricular materials and our fraction instruction, making them more informed by the existing research. A lasting improvement can only result if we engage in such critical reflection as opposed to just copying another country's approach.

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