Summary of Additional Video Excerpts (Lesson Series 2): Building Understanding of Equivalence By Connecting Fraction Strips to Number Line

Overview: The series of lessons was intended to have Dr. T. show how he would follow up on the first series of lessons, using the number line with fraction strips to go on to build students' understanding of equivalence. Dr. T. discovers that some students do not yet grasp some basic ideas about unit fractions– e.g., that one-fourth is made by dividing one whole into four equal parts, that one-fourth is the length that goes four times into 1 whole, etc. Nor have students developed strong knowledge of the number line or of fraction notation (e.g., what the denominator means). So Dr. T. devotes time to building these basic understandings, and to helping students see the number line as a tool they can use to make sense of fractions greater than one. While fluency with the number line and understanding of equivalency are only beginning to develop, the lessons richly surface many challenges in teaching fractions.

Note: One additional challenge of these lessons, which you may or may not wish to introduce, is use of a liquid measurement context (gallons, half-gallons, quarts), rather than meters. (Meters are used in the textbook and the earlier series of lessons by Dr. T.) These liquid measurement units offer a real-world context familiar to students. However, it takes some time to help students see that a "quart" is a quarter gallon, and to think of the quarter and the half gallon in relationship to the whole gallon, rather than as different wholes.

The PowerPoint Dr. T. used for these lessons at tinyurl.com/drtppt.

SHORT TALK BY DR. T. (4 minutes) at tinyurl.com/drttalk	*Using Fraction Strips and the Number Line to See Equivalence*	In this short talk, Dr. T. reflects back on the purpose of the series of four lessons: to help students connect fraction strips to the number line in order to develop their understanding of unit fractions, of mixed numbers, and of equivalence.
DAY 1: CONNECTING FRACTION STRIPS TO NUMBER LINE (22 minutes) at tinyurl.com/fractionstripday1	**Lesson Overview** Students have used fraction strips in a prior unit of instruction, but they have not connected fraction strips to a number line. This lesson connects the two representations to lay the groundwork for understanding equivalence.	A problem is introduced: Which is more: 2 gallons or 3 half-gallons? (Problem posed using containers from two teachers' families.) After students consider this problem, Dr. T. asks them to represent both quantities on the board using fraction strips (which they have created in a previous unit taught by their regular classroom teacher) and to place the fraction strips below the number line.* When asked to show $1\frac{1}{2}$ on the number line, a student points to $2\frac{1}{2}$ and another student identifies the point $1\frac{1}{2}$ as $\frac{1}{2}$. Comments of two more students help the class establish why $1\frac{1}{2}$ is where it is on the number line. Dr. T. introduces the notation $1\frac{1}{2}$ as a way to express "one gallon and a half a gallon" and explains it is a "mixed number." He also shows $\frac{3}{2}$ as the way to express three half-gallons.

		A second problem is introduced: Which is more: 2 gallons or 9 quarter gallons? (Nine quart containers are used to show nine quarter-gallons.) Students work on this problem before discussing it as a class, and then the class uses fraction strips and the number line on the board to represent the problem. A student claims that four quarter-gallons equal one gallon, but leaves gaps when demonstrating this with fraction strips, so that the number line does not reflect the equivalence of one gallon and four quarter-gallons. Students place strips representing 9 quarter gallons below the number line and count to check how many there are, reinforcing the idea of nine one-fourths. The class then revisits the problem, and Dr. T. asks students to use a "greater than" or "less than" symbol (>, <) to represent the relationship between 2 and $\frac{9}{4}$. **Note: The colors of Dr. T.'s fractions strips here in Video Series 2 differ slightly from the colors used in Video Series 1.*
DAY 3: CONNECTING FRACTION STRIPS TO NUMBER LINE Video: (10 minutes) at tinyurl.com/ fractionstripday3	**Lesson Overview** This lesson continues to use the number line and fraction strips to help students understand and compare mixed numbers and fractions greater than 1.	The lesson begins with reading journal reflections from two students. The selections emphasize that the number line can be used to compare fractions, and that 2 equals $\frac{6}{3}$. Dr. T. revisits the Day 1 lesson in which the number line and fraction strips were used to compare 2 gallons and 3 half-gallons, and gives students a representation of the problem (with number line and fraction strips) to paste in their notebooks. When he points to $\frac{3}{2}$ on the number line and asks what fraction represents it, a student incorrectly answers "$\frac{1}{2}$." Classmates show that it is $1\frac{1}{2}$ by making connections between the strips and number line. Dr. T. asks for another name for the point $\frac{3}{2}$, and a student calls it "three half-gallons" which the student then incorrectly writes as "$3\frac{3}{2}$." The class discusses this response and revisits the idea that the whole number portion of a mixed number counts wholes. Dr. T. then poses the problem: "Can you show 9 quarter gallons using a mixed number and a fraction?" This question does not get answered by the end of the lesson, because students do not all seem sure that a

		quarter is a half of a half, or that four quarters make a whole, so time is spent establishing these ideas at the board, using fraction strips and number line.
DAY 4: CONNECTING FRACTION STRIPS TO NUMBER LINE Video: trt 22 min. at tinyurl.com/ fractionstripday4	**Lesson Overview** This lesson continues to use the number line and fractions strips to help students understand and compare mixed numbers and fractions greater than 1. Along the way, it surfaces gaps in students' basic grasp of unit fractions and fraction notation and seeks to help students examine and strengthen their thinking.	The class revisits the problem of 2 gallons vs. 3 half-gallons introduced on Day 1, using pictures of gallons and half-gallons, along with fraction strips and a number line. When Dr. T. asks "what do you mean by half?" a student indicates only that it is smaller than a whole, prompting Dr. T. to rip a strip into 2 unequal parts and ask if one is a half. A student mistakenly writes three halves as $\frac{2}{3}$, and the class revisits the meaning of numerator and denominator. Next, Dr. T. shows five numbers that class members think express 9 quarters. Three are incorrect: $\frac{9}{9}$, $2\frac{1}{2}$, $9\frac{1}{4}$. Two are correct: $\frac{9}{4}$ and $2\frac{1}{4}$. One student who has (correctly) written $\frac{9}{4}$ says she has changed her mind to $2\frac{1}{2}$, whereas a classmate who answered $2\frac{1}{2}$, now says he disagrees with himself "because 4 quarters make a whole." When asked how he knows that a fraction strip is a quarter, a student says "because it's smaller than a half and even smaller than a whole," prompting Dr. T. to rip a small piece off a whole and ask if it is a quarter. Dr. T. uses the number line to show $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ and asks students to name the fractions that result, and then returns to the students' answers that they thought represented nine quarters, to ask if they are correct. Students then revisit an earlier problem of $\frac{6}{3}$, and some students still find it challenging to explain the meaning of $\frac{6}{3}$ and to create an accurate representation. One student creates a rectangle that represents $\frac{3}{6}$ (rather than $\frac{6}{3}$), another talks about dividing 6 circles each into 3 equal pieces, and another creates a number line with unequal "thirds," prompting Dr. T. again to tear a whole into 3 unequal pieces and ask if they are thirds.