TSG 19: Research and Development in Problem Solving in Mathematics Education

Beyond Show and Tell: *Neriage* for Teaching through Problem-Solving
- Ideas from Japanese Problem-Solving Approaches for Teaching Mathematics -

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Abstract

Japanese teachers use problem solving as a powerful approach for teaching mathematics. There are several notable characteristics of the Japanese approach to problem solving. One of the characteristics is that Japanese problem solving lessons usually do not end even after each student finds a solution to the problem. Japanese teachers and researchers believe that the heart of the lesson begins *after* students come up with solutions. The teacher facilitates extensive discussion with students, which is called *Neriage,* by comparing and highlighting the similarities and differences among students' solutions. In this paper, some of the characteristics of the Japanese approach for teaching mathematics will be discussed by focusing on the heart of the approach, *Neriage*.

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Introduction

Problem solving has been a major focus in Japanese mathematics curricula for nearly a half century. Numerous teacher reference books and lesson plans using problem solving have been published since the 1960s. Government authorized mathematics textbooks for elementary grades, which are published by six private companies, have had more and more problem solving over the years. As a result, almost every chapter in recent Japanese mathematics textbooks for elementary grades begins with problem solving as a way to introduce new concepts and ideas to students.

A few key publications have greatly influenced how problem solving is used in Japanese mathematics education. Polya's *How to Solve It* (Polya, 1945, p. 26) was translated and published in Japanese in 1954, and had been studied by various researchers and educators in Japan. Japanese researchers, teachers, and administrators worked collaboratively through Lesson Study, a professional development approach that is popular in Japan, to develop mathematics instruction by referring to Polya's (1945) four phases of problem solving work (Takahashi, 2000). One of the results from the studies of problem solving, *Open-ended Approach,* was published in 1977 by Shimada et al. The open-ended approach has been widely used in Japanese classrooms since then. Moreover, the English translation of the book was published (J. P. Becker & Shimada, 1997) and has been popular among educators in the U.S. too. The Ministry of Education in Japan has recognized the importance of problem solving in school mathematics and emphasized the need for students to develop problem-solving skills to learn and use mathematics in various documents since the beginning of the 1980s. The position statement from the NCTM's *An agenda for action: Recommendations for school mathematics of the 1980s* (1980) that "problem solving must be the focus of school mathematics" was referenced in various research articles and resource materials for teachers in Japan during the 1980s. Also, *Teaching Problem Solving: What, why & how* (Charles & Lester, 1982) was translated into Japanese in 1983.

Stigler and Hiebert (1999) described Japanese mathematics lessons as "structured problem solving". Similar characteristics of Japanese mathematics lessons were also reported in the proceedings of the U.S.-Japan Seminar of Mathematical Problem Solving (Jerry P. Becker & Miwa, 1987; Jerry P Becker, Silver, Kantowski, Travers, & Wilson, 1990). Structured problem solving is designed for students to acquire knowledge and skills through creative mathematical activity by presenting challenging problems to students. Students are expected to solve a problem

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using their own mathematical knowledge. Thus, Japanese teachers usually do not tell students how to solve a problem before students try to solve the problem by themselves. Working with problems, students bring several different approaches and solutions to the class. The teacher then leads students in a whole-class discussion in order to compare individual approaches and solutions. This whole-class activity provides students with opportunities to learn mathematics. Through their extensive study of problem solving, Japanese teachers and educators have come to recognize that this whole-class discussion is the heart of structured problem solving and have named this discussion part *Neriage.*

In this paper, I will discuss the Japanese approach of using problem solving for teaching mathematics, structured problem solving, by focusing on the heart of the approach, *Neriage*.

The Japanese Problem Solving Approach

One of the major goals of teaching mathematics is to help students become able to solve problems. Thus, mathematics lessons employing problem solving are sometimes viewed as an approach for students to develop problem-solving skills and strategies, and teachers sometimes focus solely on the strategy of solving the problem and not necessarily on developing mathematical concepts and skills. This interpretation of problem solving lessons usually ends after each student comes up with a solution to the problem. The teachers' role during students' problem solving is to help students find the solution by providing an efficient strategy, because the major goal of the lesson is for students to solve problems.

On the other hand, problem solving can also be viewed as a powerful approach for developing mathematical concepts and skills. Thus, in this approach teachers use problem solving not only for lessons that solely focus on developing problem-solving skills and strategies but also on lessons that develop mathematical concepts, skills, and procedures. As a result the lesson plans for this approach usually include content goals in addition to goals for developing problem solving strategies and skills.

To highlight the difference between the two approaches, the latter approach is often called "teaching though problem solving." Since Japanese structured problem solving uses problem solving as a process for learning mathematical content, it could be considered a type of teaching through problem solving. Looking at problem solving as a process for students to learn mathematics is not unique to Japanese mathematics education. The National Council of Teachers of Mathematics (NCTM) has also emphasized the importance of teaching mathematics through

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problem solving (Mathematics, 2006; National Council of Teachers of Mathematics, 1989, 2000, 2006). These documents discuss the necessity of learning mathematical content through the processes of problem solving, reasoning and proof, communication, connections, and representation. Various reform documents have also suggested that mathematics lessons should be designed to provide students learning opportunities through the processes of problem solving and not simply by listening to teachers' lectures.

Although teaching through problem solving has been suggested by NCTM and other reform documents in the U.S., it is hard to find lessons that employ this idea in U.S. classrooms. Stigler and Hiebert argue that Japanese mathematics lessons better exemplify current U.S. reform ideas, such as teaching through problem solving, than do typical U.S. mathematics lessons, based on the TIMSS videotape classroom study (1997). One of the reasons behind this phenomenon might be that Japanese mathematics teaching already had a history of focusing on developing mathematical thinking skills by using a variety of story problems even before the idea of problem solving was introduced. Therefore Japanese educators looked at problem solving as an ideal approach for learning mathematics rather than simply a way to promote problem solving skills when the idea of problem solving was introduced.

There are several notable characteristics of the Japanese problem solving approach. First, the Japanese problem solving approach can be found throughout the curriculum because it is designed for learning mathematical content. Japanese teachers have tried to use the approach for students not only to develop concepts and understanding of mathematics but also to acquire skills to learn and use mathematics. Therefore problem solving is not viewed as an end-of-the-chapter activity that is solely focused on developing problem-solving skills and strategies. Second, Japanese problem solving lessons usually do not end even after each student finds a solution to the problem. Problem solving lessons that solely focus on developing problem solving and skills often end after students share their solutions with the class. Japanese teachers and researchers, however, believe that the heart of the lesson begins after each student comes up with a solution(s). Japanese teachers facilitate extensive discussion with students, which is called *Neriage,* by comparing and highlighting the similarities and differences among students' solution approaches.

Extensive discussion (*Neriage*)

The term *Neriage* has been widely used among Japanese teachers and researchers of mathematics education as a technical term since 1980s. *Neriage* is a noun form of a verb *Neriageru*, which means to polish up. The term *Neriage* is used among Japanese teachers for describing the dynamic and collaborative nature of a whole-class discussion in the lesson (Shimizu, 1999). The most important role of the teacher in *Neriage* is to orchestrate students' ideas and approaches to solve the problem and to help them polish their solutions in order to learn mathematical content. During the process, a teacher highlights important mathematical ideas and concepts for students to reach the goals of the lesson. This is why Japanese teachers see *Neriage* as the heart of teaching mathematics through problem solving. From the viewpoint of Japanese teachers, the solving of the problem by each student at the beginning of the lesson is a preparation for *Neriage*. Therefore it is important for students to struggle with the problem and find their own way to solve the problem, because this experience will be the foundation for students to make a connection between their previous learning and the content that they are going to learn through *Neriage*.

The following is an example of a series of problem solving activities from the most

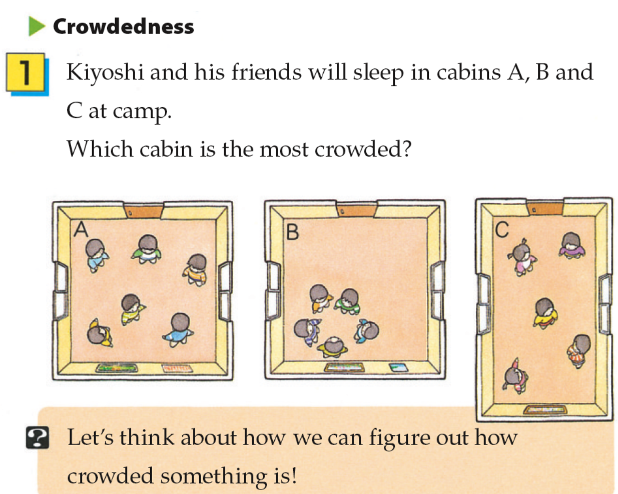

Figure 1. Which cabin is the most crowded?
Reprinted from *Mathematics for Elementary School 5B*. Tokyo, Japan: Tokyo Shoseki Co., Ltd. p.23 (Hironaka & Sugiyama, 2006)

widely used Japanese mathematics textbook series. This example shows how *Neriage* leads students to acquire a new idea in mathematics through a series of problem solving activities. The

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unit on Per Unit Quantity in the 5th grade textbook begins with the following problem (Figure 1) without showing any solution or hint to the students on the page. The textbook is designed for students to see everyday situations as mathematical problems. Most 5th grade students may realize that the crowdedness of Cabin A and Cabin B may be compared by looking at the number of the people who share each cabin and because the areas of the rooms look the same. Students may also be able to see that the crowdedness of Cabin B and Cabin C may be compared by looking at their area because the number of the people who share each cabin are the same. The textbook expects teachers to facilitate the above discussion at the beginning of the lesson so that students understand the situation. Then, the next page of the textbook (Table 2) provides further information for students to understand what is the mathematical problem in this everyday situation.

	Area (m^2)	Number of people
Cabin A	16	6
Cabin B	16	5
Cabin C	15	5

Table 2. Which cabin is the most crowded?
Reprinted from *Mathematics for Elementary School 5B*. Tokyo, Japan: Tokyo Shoseki Co., Ltd. p.24 (Hironaka & Sugiyama, 2006)

By looking at the data, students are able to compare the crowdedness of Cabin A and Cabin B without any calculation. They also may be able to see which room is more crowded between Cabin B and Cabin C. It is, however, not easy for students to figure out which is more crowded between Cabin A and Cabin C. Through this discussion, teachers are expected to lead students to understand what is the problem to be solved. There are two data for each cabin, the area of the cabin and the number of people who share the cabin. It is easy to compare the crowdedness of the cabins if one of these two data are the same. Since neither the area nor the number of people for Cabin A and Cabin C are the same, it might not be possible to compare the crowdedness by using the data. If so, are there any ways to make one of the data the same -- either the area or the number of the people?

After the above discussion, each student is encouraged to figure out which is more crowded, Cabin A and Cabin C. Since the numbers in the problem are carefully chosen, there are several approaches for students to compare crowdedness. The following are four commonly seen student solution methods.

Method A:

Cabin A: $6 \div 16 = 0.375$

Cabin B: $5 \div 15 = 0.33\cdots$

Division is used to find how many people occupy 1 m^2. Because a larger number of people would occupy 1 m^2, Cabin A is more crowded.

Method B:

Cabin A: $16 \div 6 = 2.66\cdots$

Cabin B: $15 \div 5 = 3$

Division is used to find how many square meters there are per person. Because there is less area per person, Cabin A is more crowded.

Method C:

A common multiple of 16 and 15 is 240

Cabin A: $6 \times 15 = 90$

Cabin B: $5 \times 16 = 80$

This method looks at how many people would share each cabin if both cabins have the same area. In order to use this method, a common multiple 240 is found as the area of each cabin. Because more people would share 240 m^2, Cabin A is more crowded.

Method D:

A common multiple of 6 and 5 is 30

Cabin A: $16 \times 5 = 80$

Cabin B: $15 \times 6 = 90$

This method looks at how much area would be shared by a person if both cabins have the same area. In order to use this method, a common multiple 30 has been found as the number of people in each cabin. Because less area would be occupied by a person, Cabin A is more crowded.

After students come up with solution methods that include the above four methods, the teacher assigns students to share their solutions. Japanese teachers usually monitor students' work during their individual or group problem-solving time and come up with a plan for the discussion. For example, teachers often use a seating chart of the class to jot down how each student approaches the problem as well as to plan how to lead the discussion. A teacher might ask one of the students who used the most common method to share his/her method to begin the discussion, and then ask another student to share different methods. At this time, Japanese teachers usually do not say that the answers are right or wrong in order to provide students with opportunity to think carefully about each solution method. Teachers carefully use blackboard writing for students to see all the solution methods from their peers and to help them understand each method.

The *Neriage* begins after the students present their various solution methods. Until *Neriage* begins, the whole class activity is very similar to the children's favorite school activity, Show and Tell. *Neriage* is an activity that goes beyond Show and Tell, however. If the goal of the problem-solving lesson is just to find a solution to the problem, this could be the end of the lesson. But because the Japanese problem-solving lesson is designed for students to learn new mathematical knowledge, the *Neriage* is necessary.

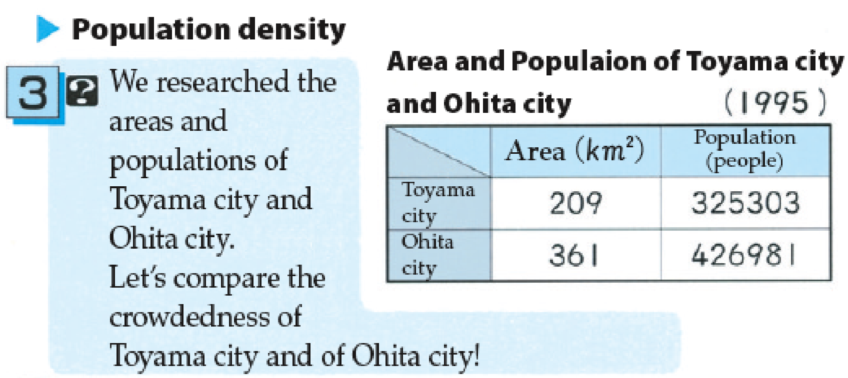
Teachers might begin the *Neriage* by asking students to see if there are some common ideas or approaches among the solution methods or some differences. For example, students might notice that both Method A and Method B use division but in different ways. In contrast Method C and Method D use multiplication instead of division. On the other hand, Method A and Method C use the same idea that looks at what if both cabins had the same area. Method B and Method C also share the same idea that looks at what if both cabins were shared by the same number of people. During the discussion, teachers would provide opportunities to think about why using addition and subtraction might not be the best approach to this problem. This comparison would help students deepen their understanding of the concept and the use of multiplication and division concerning ratio and rate. If there are some methods based on students' misunderstanding, teachers can use them to help students develop reasoning skills to justify whether the solutions are right.

Then, teachers would lead students to see whether each approach has advantages and limitations. Students might realize that the methods that use a common multiple might have a limitation if the number in the problem becomes larger or if the situation requires them to compare more than two rooms. Finding a common multiple with large numbers or with several numbers might not be an easy task. Considering this limitation, teachers might be able to conclude that the use of division to find a unit quantity might be a better way for the similar problem with a more complex situation. In fact, the Japanese textbook series gives students the following problem after “Which cabin is the most crowded?” for students to actually see which method, multiplication or division, is most useful.

> At Yoshiko’s farm, which is 600m^2, 1968kg of potatoes were produced. At Tadashi’s farm, which is 900m^2, 2682kg of potatoes were produced. Which farm was better at producing potatoes? (Hironaka & Sugiyama, 2006)

Teachers would also lead students to see whether there are any advantages to Method A and Method B. Comparing these two methods requires students to think deeply about the meaning of division. Moreover, this opportunity can help students to understand practical application of the use of division, since the order of division requires students to use different interpretations of the quotients. The quotients in Method A show that the larger quotient indicates more crowdedness. The quotients in Method B show that the larger quotient indicates less crowdedness. To use a number to represent crowdedness, the quotients in Method A are more understandable, because the larger number means that it is more crowded. Later in the unit, this Japanese textbook introduces population density, comparing crowdedness by using the number of people who live in an area of 1km^2, and asking students to solve the problem.

Neriage is a critical component of a Japanese problem-solving lesson because this is the place where teachers can teach students new mathematical ideas and concept by using students’ solution methods. Because the *Neriage* is built upon the students’ solutions as a foundation of the


Population density

3 We researched the areas and populations of Toyama city and Ohita city. Let’s compare the crowdedness of Toyama city and of Ohita city!

Area and Populaion of Toyama city and Ohita city (1995)

	Area (km^2)	Population (people)
Toyama city	209	325303
Ohita city	361	426981

Figure 2. Population dencity
Reprinted from *Mathematics for Elementary School 5B*. Tokyo, Japan: Tokyo Shoseki Co., Ltd. p.27 (Hironaka & Sugiyama, 2006)

dynamic and collaborative whole-class discussion, Japanese teachers put so much effort to preparing the discussion.

Lesson Planning for Problem Solving Lesson: Beyond Show and Tell

At the beginning of the development of the Japanese problem solving approach, the term *Neriage* had not been widely used. Instead, educators used the Japanese translation of terms from Polya's four phases of problem solving—understanding the problem, devising a plan, carrying out the plan, and looking back—as a framework to design lessons. For example, 1) the problem solving lesson usually began with the presentation of the problem of the day by the teacher and the teacher helping students to understand what the problem really is, thus, "understanding the problem", 2) then the teacher asked each student to devise a plan to solve the problem, thus "devise a plan", 3) next, students solved the problem by using their previously learned knowledge and skills, thus "to solve the problem based on the plan," and finally, 4) students examined whether their solution was correct and the method that they used was reasonable and efficient, thus, "looking back." Although Polya's four phases of problem solving had been used as the foundation of Japanese problem solving lessons, Japanese teachers revised the framework over the years through lesson study. There are two notable changes over the years. First, the 2nd phase, devise a plan, was omitted from the flow of the lesson. Second, to describe the 4th phase, looking back, the term *Neriage* began to be used. These changes did not happen in a top-down manner. Through numerous research lessons and post-lesson discussions, teachers gradually shifted their use of problem solving and reached the problem solving approach which Stigler and Hiebert (1999) described as structured problem solving.

Teaching mathematics through problem solving is not an easy task for teachers, especially facilitating a good discussion, *Neriage*. To develop problem-solving lessons, Japanese teachers usually begin by considering the following three major issues—the curriculum, the students, and the problem. By investigating the curriculum teachers should be able to identify what content should be taught. Japanese teachers try to identify the contents as specific as possible so that the lesson will be focused on a specific topic(s). Then, they examine students' previous learning to identify the goal of the problem-solving lesson and to identify the problem for the lesson. Japanese teachers seek meaningful problems for problem solving lessons by looking at Japanese mathematics textbooks and resources materials. These resource materials include lesson study reports and books published by experienced lesson study practitioners. These three issues can be discussed in any order.

Sometimes Japanese teachers develop lessons for lesson study from scratch but they often develop lessons by using others' work and modify them to fit into their own students' needs.

One of the challenges is to find a good problem that can lead students to accomplish the goals of the lesson. There are a lot of interesting and engaging problems, including puzzles and games; however, the problem should be able to foster students' ability to learn something new after they have solved the problem by using their existing knowledge and skills. In other words, when students solve the problem it is expected that the problem provides students with opportunities for see a need for learning new knowledge and skills, which is the goal of the lesson.

A group of teachers carefully examine problems for students. They always solve each problem by themselves in several different ways in order to examine whether the problem is mathematically meaningful for the students at the time of the lesson. Then, teachers begin to anticipate students' responses to the problem including ones based on misunderstandings and misuses of previous learning. Then they start to design the flow of the lesson so that students will be able to reach the goal of the lesson through problem-solving. Coming up with good questioning for students to think deeply about mathematics is always a challenge for teachers when designing lessons.

One of the major tasks for Japanese teachers is to facilitate meaningful mathematical discussion during the whole-class activity to help students to achieve the goals of the lesson. When a teacher presents a problem to students without giving a procedure, it is natural that several different approaches to the solution will come from the students. Thus, the textbooks include examples of students' typical approaches and ideas. Because the goal of the structured problem-solving approach is to develop students' understanding of mathematical concepts and skills, a teacher is expected to facilitate mathematical discussion for students to achieve this goal. This discussion is often called *Neriage* in Japanese, which implies polishing ideas. In order to do this, teachers need a clear plan for the discussion as a part of their lesson plans, which will anticipate the variety of solution methods that their students might bring to the discussion. These anticipated solution methods include not only the most efficient methods but also ones caused by students' misunderstandings. Thus, anticipating students' solution methods is a major part of lesson planning for Japanese teachers.

Towards the end of a lesson, a teacher often leads the lesson in a way that pulls all the different approaches and ideas together in order to see the connection. Then, he or she summarizes the lesson to help students achieve the objective of the lesson. The teacher often asks students to reflect on what they have learned during the lesson.

Japanese teachers and researchers believe that *Neriage* is the most important component of teaching mathematics through problem solving. Therefore, teachers spend much time investigating various resources through *Kyozaikenkyu* in lesson study (Watanabe, Takahashi, & Yoshida, in press). The quality of *Neriage* depends upon how well the teacher(s) plan the lesson, because this is the place where teachers have to use all their knowledge of mathematics, their knowledge about teaching mathematics, their knowledge of students, and their skills to facilitate the whole-class discussion. Therefore, Japanese educators believe that teaching, especially in *Neriage*, is the proving ground of teachers' knowledge and skills (Fujii, in press).

Conclusion

This paper focuses on the term *Neriage* to highlight the characteristics of a Japanese teaching approach based on problem solving. There are several other technical terms in the Japanese educational community. These technical terms are sometimes used among teachers to describe and discuss specific events or techniques in teaching and learning. This means that a term such as *Neriage* is used only among teachers, and people in other professions do not share the meaning of the term. The existence of such technical terms tells us that Japanese teachers have an opportunity to discuss teaching and learning with their colleagues regularly.

The Japanese teaching profession has a long history of collaboration among teachers to discuss how to improve teaching and learning. The Japanese problem-solving approach is one of the outcomes of this collaboration. Thus, the development of collegial communities among teachers is an ideal way to make a shift in the teaching and learning of mathematics by using suggestions from various documents.

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